

Decay Rate Statistics of Unstable Classically Chaotic Systems

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Abstract

Decay law of a complicated unstable state formed in a high energy collision is described by the Fourier transform $K(t)$ of the two-point correlation function of the scattering matrix. Although each constituent resonance state decays exponentially the decay of a state composed of a large number $N \gg 1$ of such interfering resonances is not, generally, exponential. We introduce the decay rates distribution function $w(\Gamma)$ by representing the decay law in the form of the mean-weighted decay exponent $K(t) = \int_0^\infty d\Gamma e^{-\Gamma t} w(\Gamma)$. In the framework of the random matrix approach we investigate the properties of the distribution function $w(\Gamma)$ and its relation to the more conventional statistics of the decay widths. The latter is not in fact conclusive as concerns the evolution at the times shorter than the characteristic Heisenberg time. Exact analytical consideration is presented for the case of systems without time reversal symmetry.

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I. INTRODUCTION

The temporal aspect of chaotic resonance scattering is an interesting and important issue which repeatedly attracted much attention starting from the seminal works of Wigner and Smith [1, 2]. Later on different problems concerning the duration of resonance collisions were posed and discussed in detail. Distribution of resonance widths on the one hand and statistics of partial and proper delay times on the other have been investigated in a number of papers [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16].

It is generally known that a single quasi-stationary state formed by the time $t = 0$ in a collision with a given energy E decays afterwards exponentially with the decay rate Γ_r which is given by the imaginary part of the resonance complex energy $\mathcal{E}_r = E_r - \frac{i}{2}\Gamma_r$. In accordance with the Bohr energy-time uncertainty principle this rate defines the width of the Breit-Wigner resonant curve.

However at high enough collision energy E a very complicated unstable state is formed which is a superposition of many, $N \gg 1$, interfering resonance contributions whose spectrum is given by the N complex eigenvalues $\mathcal{E}_r = E_r - \frac{i}{2}\Gamma_r$ of $N \times N$ matrix of the non-Hermitian effective Hamiltonian $\hat{\mathcal{H}} = \hat{H} - \frac{i}{2}\hat{W}$, $\hat{W} = \hat{A}\hat{A}^\dagger$. The resonances, generally, overlap so that their mean width is larger than the mean level spacing, $\langle\Gamma\rangle > \Delta$. The evolution of such an unstable configuration formed via an incoming channel a and decaying onto an outgoing channel b is described [4, 7] by the Fourier transform

$$K^{ba}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega e^{i\omega\tau} C^{ba}(\omega) \quad (1)$$

of the two-point S-matrix correlation function

$$C^{ba}(\omega = t_H \varepsilon) = \frac{\langle S^{ba}(E + \varepsilon/2) S^{ba*}(E - \varepsilon/2) \rangle_{conn}}{T^b T^a}; \quad b \neq a \quad (2)$$

Here $T^c = 1 - |S^{cc}|^2$ is the transmission coefficient in the channel c and $t_H = 2\pi/\Delta$ is the Heisenberg (Weisskopf) time. It is convenient to measure the time in the units of this characteristic time interval, $\tau = t/t_H$. Correspondingly, the dimensionless energy shift $\omega = \varepsilon t_H$ is introduced in the Eqs. (1, 2). To avoid complications irrelevant in the present context we restrict for a while our consideration to inelastic collisions only.

Contrary to the case of an isolated quasi-stationary state, the function $K^{ba}(\tau)$ does not, generally, decays exponentially and a connection to the widths of the constituting resonances

is not immediately seen. More than that, an opinion has not once been expressed that the very notion of the resonance widths becomes irrelevant when resonances strongly overlap. Nevertheless, intuitively, one would expect that decay properties of a complicated unstable state should somehow depend on the distribution of the individual resonance poles of the scattering amplitudes in the complex energy plane. Could then a non-exponential decay results from an incoherent mixture of exponential decays with different rates? If and how such a mixture is related to the statistics of the widths of resonances? We address these questions below. They are answered explicitly by the example of the chaotic systems without time-reversal symmetry. A rigorous analytical solution is found in this case.

II. DECAY RATES DISTRIBUTION FUNCTION

The function $C^{ba}(\omega)$ is analytical in the lower part of the complex ω plane and, obviously, satisfies the condition $C^{ba}(-\omega) = C^{ba*}(\omega)$. As a result its Fourier transform $K^{ba}(\tau) \equiv 0$ when $\tau < 0$ and is real when $\tau > 0$. More than that, independently of the number M of the reaction channels, [17]

$$K^{ba}(0) = 1 \quad \text{and} \quad K^{ba}(\tau) > 0 \quad \text{when} \quad \tau > 0. \quad (3)$$

Notice at last that the mean reaction cross section is expressed as

$$\langle \sigma^{ba} \rangle = T^b T^a C^{ba}(0) = T^b T^a \int_0^\infty d\tau K^{ba}(\tau). \quad (4)$$

Such properties suggest the Laplace representation for the decay law function

$$K^{ba}(\tau) = \int_0^\infty d\gamma w^{ba}(\gamma) e^{-\gamma\tau}. \quad (5)$$

(In the ordinary units $\gamma = \Gamma t_H = \frac{2\pi}{\Delta} \Gamma$ where Γ is the decay rate.) In view of the Eq. (3) the function w^{ba} is normalized to unity,

$$\int_0^\infty d\gamma w^{ba}(\gamma) = K^{ba}(0) = 1. \quad (6)$$

It follows from the Eqs. (1, 5) that

$$C^{ba}(\omega) = \int_0^\infty d\gamma \frac{w^{ba}(\gamma)}{\gamma + i\omega} \quad (7)$$

so that the correlation function is not, generally, single-valued in the complex ω plane. Finally, according to the Eqs. (4, 7),

$$\langle \sigma^{ba} \rangle = T^b T^a \int_0^\infty d\gamma \frac{w^{ba}(\gamma)}{\gamma}. \quad (8)$$

From this point on we suppose that all channels are statistically equivalent, $T^b = T^a = T$, so that the functions $C(\omega)$, $K(\tau)$, $w(\gamma)$ do not depend on the channel indices.

III. THE SEMICLASSICAL ASYMPTOTIC EXPANSION

Let us consider first the semiclassical limit: $N \gg 1$, $M \gg 1$ but the ratio $m = M/N$, is finite though small, $m \ll 1$, (this condition is physically justified). The function $w(\gamma)$ can easily be extracted from the results reported in: [5, 9] (GOE)

$$C(\omega) = \frac{1}{MT} \frac{\gamma(\omega)}{\gamma(\omega) + i\omega} \Rightarrow \frac{1}{\gamma_W + i\omega} - \frac{2T}{(\gamma_W + i\omega)^2} + \frac{MT^2}{(\gamma_W + i\omega)^3} + \dots \quad (9)$$

where $\gamma(\omega)$ is a slow varying function and $\gamma(0) \equiv \gamma_W = MT = t_H \Gamma_W$ where $\Gamma_W = \frac{\Delta}{2\pi} MT$ is the Weisskopf width. The asymptotic series in the r.h.s which is obtained by expanding near the pole $\omega = i\gamma_W$ coincides with that derived first in different manner in: [18]. The first term corresponds to the Hauser-Feshbach approximation and implies the purely exponential decay $e^{-\gamma_W \tau}$ when the subsequent ones account for deviations. In the limit $N, M \rightarrow \infty$, $0 < m \ll 1$ the Weisskopf width equals to the empty gap between the real energy axis and the cloud of the poles corresponding to contributing resonances [9]. The width Γ_W is in this case the smallest possible width.

It immediately follows from the expansion (9) that

$$w(\gamma) = \delta(\gamma - \gamma_W) - 2T \delta'(\gamma - \gamma_W) + \frac{MT^2}{2} \delta''(\gamma - \gamma_W) + \dots \quad (10)$$

The function $w(\gamma)$ thus obtained is neither smooth nor positive definite. We will see below that the expansion (9) is in this respect somewhat misleading. Nevertheless it works well in certain cases. In particular, for the mean cross section we get from the Eqs. (8, 10)

$$\langle \sigma \rangle^{(GOE)} = \frac{T}{M} \left(1 - \frac{1}{M} + \dots \right). \quad (11)$$

Similar calculation yields in the GUE-case $\langle \sigma \rangle^{(GUE)} = \frac{T}{M}$.

Notice that the difference $\langle \sigma \rangle^{(GUE)} - \langle \sigma \rangle^{(GOE)} = \frac{T}{M^2}$ determines the "weak localization" in the quantum transport.

IV. DECAY RATES VERSUS WIDTHS STATISTICS

Heuristic" arguments have been adduced in [19] which assume a simple connection between the function $w(\gamma)$ and the distribution $\rho_M(\gamma)$ of the resonance widths

$$w_M(\gamma) = A \gamma^2 \rho_M(\gamma) \quad (12)$$

with some normalization constant A . A formula of such a kind has been derived a little later in [8]. It was shown, however, that the relation supposed can reproduce only the long-time asymptotic behavior. Only the most long-lived resonances survive by this time, so they can be expected to obey the χ_M^2 widths statistics.

$$\rho_M(\gamma) = \frac{1}{(\beta \frac{M}{2})!} \frac{1}{\gamma} \left(\beta \frac{\gamma}{2T} \right)^{\beta \frac{M}{2}} e^{-\beta \frac{\gamma}{2T}}, \quad \langle \gamma \rangle = MT = \gamma_W, \quad \beta = 1, 2. \quad (13)$$

Here the parameter β marks the Dyson's symmetry class: GOE ($\beta = 1$) or GUE ($\beta = 2$). Notice that in our dimensionless units $\Gamma/\langle \Gamma \rangle = \gamma/MT$ which means that $\langle \gamma \rangle = MT = \gamma_W$. In the case of isolated resonances the Weisskopf width defines the mean rather than the minimal width. The formal extending the relation (12) to all values of γ yields the result

$$K_M(\tau) = A M \left(M + \frac{\beta}{2} \right) \frac{T^2}{\left(1 + \frac{2T}{\beta} \tau \right)^{\beta \frac{M}{2} + 2}} \rightarrow \frac{const.}{\tau^{\beta \frac{M}{2} + 2}} \quad (14)$$

which reproduces rightly the power law asymptotic behavior [5] though the proper calculation of the constant is beyond the validity of this approximation. The constant A can be chosen to quantitatively fit the correct asymptotics. However at shorter times the found in such a way expression disagree with the actual decay law, the discrepancy being the stronger the closer is the transmission coefficient T to its maximal value 1 (see [8]). There is no way to reconcile all necessary conditions which must be satisfied by the decay function $K_M(\tau)$ by means of only one matching parameter A . There are two possible reasons why the supposed relation (12) fails: i. the χ_M^2 distribution is not valid when resonances overlap; ii. the naive derivation is wrong. We will clarify below these guess-work by the example of the systems with broken time-reversal symmetry which are described by the Gaussian unitary ensemble (GUE) of random Hamiltonians.

V. SYSTEMS WITH NO TIME-REVERSAL SYMMETRY

A. General consideration

The GUE two-point correlation function derived in: [20] reduces in the case of equivalent channels to

$$C(\omega) = \int_0^1 d\lambda \int_0^\infty d\lambda_1 e^{-i\omega(\lambda+\lambda_1)} \frac{1}{\lambda+\lambda_1} \frac{(1-T\lambda)^{M-2}}{(1+T\lambda_1)^{M+2}} \times \left\{ \left(\frac{2}{T} - 1 \right) (1-T\lambda)(1+T\lambda_1) - \frac{1-T}{T} [(1-T\lambda) + (1+T\lambda_1)] \right\}. \quad (15)$$

Making use of the following simple identity

$$\frac{1}{\lambda_1 + \lambda} = \int_0^\infty d\gamma e^{-\gamma(\lambda_1 + \lambda)} \quad (16)$$

we factorize the integrations over λ and λ_1 . On the next step we use a second identity

$$\int_0^\infty d\lambda_1 \frac{e^{-(\gamma+i\omega)\lambda_1}}{(1+T\lambda_1)^{M+1+\mu}} = \frac{1}{(M+\mu)!} \int_0^\infty d\eta \frac{\eta^{M+\mu} e^{-\eta}}{T\eta + \gamma + i\omega}, \quad \mu = 0, 1. \quad (17)$$

The correlation function (15) is thus reduced to a linear combination of terms like

$$c_{\mu\mu'}(\omega) = \frac{1}{(M+\mu)!} \int_0^\infty d\gamma \int_0^\infty d\eta \eta^{M+\mu} e^{-\eta} \int_0^1 d\lambda (1-T\lambda)^{M-1-\mu'} e^{-\gamma\lambda} \frac{e^{-i\omega\lambda}}{T\eta + \gamma + i\omega}. \quad (18)$$

After a chain of simple transformations of the integration variables the Fourier transform of the latter function reduces finally to

$$k_{\mu\mu'}(\tau) = \frac{1}{T^{2+\mu+\mu'}(M+\mu)!} \int_0^\infty d\gamma e^{-\gamma\tau} \int_0^\gamma d\gamma' \gamma'^{(\mu+\mu')} \times \int_{\frac{1-T}{T}\gamma'}^{\gamma'/T} d\nu \Theta\left(\tau - \frac{1}{T} + \frac{\nu}{\gamma'}\right) \nu^{M-1-\mu'} e^{-\nu}. \quad (19)$$

The symbol Θ stands for the step function. All successive integrations in this expression can be carried out explicitly [21].

To simplify further consideration we restrict our calculation to the case of the perfect coupling to the continuum, $T = 1$, when the naive consideration is the least satisfactory. Only the term with $\mu = \mu' = 0$ remains in this limit and

$$C_M(\omega) = \frac{1}{M!} \int_0^\infty d\gamma \int_0^\infty d\gamma' \gamma'^M e^{-\gamma'} \int_0^1 d\lambda (1-\lambda)^{M-1} e^{-\gamma\lambda} \frac{e^{-i\omega\lambda}}{\gamma + \gamma' + i\omega}. \quad (20)$$

The subscript M explicitly indicates the number of open channels. Notice that when $T = 1$ the formula (20) and so all consequent results are valid also for the elastic collisions including the purely elastic process with one open channel $M = 1$.

The found representation enables us to evaluate the mean cross section in the following simple and elegant way:

$$\langle \sigma \rangle = C_M(0) = \frac{1}{M!} \int_0^\infty d\gamma F_M(\gamma) \Phi_{M-1}(\gamma) = \frac{1}{M} \int_0^\infty d\gamma e^{-\gamma} F_0(\gamma) = \frac{1}{M}. \quad (21)$$

The third equality follows from the fact that the functions

$$F_M(\gamma) = \int_0^\infty d\eta \eta^M \frac{e^{-\gamma\eta}}{1+\eta}, \quad \Phi_{M-1}(\gamma) = \int_0^\gamma d\zeta (\gamma - \zeta)^{M-1} e^{-\zeta} \quad (22)$$

which appear in this equation obey the simple recursions

$$F_M(\gamma) = -\frac{d}{d\gamma} F_{M-1}(\gamma), \quad \frac{d}{d\gamma} \Phi_M(\gamma) = M \Phi_{M-1}(\gamma). \quad (23)$$

The decay function looks now as

$$K_M(\tau) = \int_0^\infty d\gamma e^{-\gamma\tau} \frac{1}{M!} \int_0^\gamma d\gamma' \int_0^{\gamma'} d\nu \Theta(\tau - 1 + \nu/\gamma') \nu^{M-1} e^{-\nu}. \quad (24)$$

The later consideration depends on whether the time τ exceeds the Heisenberg time $\tau_H = 1$ or not.

B. Long-time asymptotics

After the Heisenberg time, $\tau \geq 1$, the step function in the Eq. (24) equals to one in the whole integration region and the decay function gets the required form

$$K_M(\tau > 1) = \int_0^\infty d\gamma e^{-\gamma\tau} w_>(M, \gamma) \quad (25)$$

with the decay rates distribution function given by

$$w_>(M, \gamma) = \frac{1}{M!} \int_0^\gamma d\gamma' \varphi_{M-1}(\gamma'), \quad (26)$$

$$\varphi_{M-1}(\gamma) = \int_0^\gamma d\nu \nu^{M-1} e^{-\nu} = (M-1)! - \Gamma(M, \gamma)$$

where $\Gamma(M, \gamma)$ is the incomplete Γ function

$$\Gamma(M, \gamma) = (M-1)! e^{-\gamma} \sum_{m=0}^{M-1} \frac{\gamma^m}{m!}. \quad (27)$$

The next integration over γ' yields finally

$$w_>(M, \gamma) = \frac{\Gamma(M+1, \gamma) - \gamma \Gamma(M, \gamma)}{M!} + \gamma \frac{1}{M} - 1 = \tilde{w}_M(\gamma) - \left(1 - \gamma \frac{1}{M}\right), \quad (28)$$

$$\tilde{w}_M(\gamma) = e^{-\gamma} \sum_{m=0}^{M-1} \left(1 - \frac{m}{M}\right) \frac{\gamma^m}{m!} \equiv e^{-\gamma} P_{(M-1)}(\gamma).$$

As expected, this distribution is smooth and positive definite. Notice that, due to subtraction of the last two terms, the distribution $w_{>}(M, \gamma)$ vanishes as γ^2 when $\gamma \rightarrow 0$.

At last the final integration over γ yields

$$K_M(\tau > 1) = \int_0^\infty d\gamma e^{-\gamma\tau} w_{>}(M, \gamma) = \frac{1}{M} \frac{1}{\tau^2 (1 + \tau)^M}, \quad K_M(\tau \rightarrow \infty) = \frac{\langle \sigma \rangle}{\tau^{(M+2)}} \quad (29)$$

thus fixing not only the power but also the constant of asymptotic power behavior.

The found results are closely connected to statistics of the decay width. Indeed the widths distribution function in the case of M equivalent channels, arbitrary degree of resonance overlapping and perfect coupling to the continuum is currently well known to be [11]

$$\rho_M(\gamma) = \frac{1}{(M-1)! \gamma^2} \int_0^\gamma d\nu \nu^M e^{-\nu} \quad (30)$$

and has nothing to do with the χ_M^2 -distribution (13). It is immediately seen that

$$\varphi_{M-1}(\gamma) = (M-2)! \gamma^2 \rho_{M-1}(\gamma) \quad (31)$$

(see the Eq. (26)). This results in the following connection between the two distributions

$$\begin{aligned} w_{>}(M, \gamma) &= \frac{1}{M(M-1)} \int_0^\gamma d\gamma' \gamma'^2 \rho_{M-1}(\gamma') = \\ \frac{1}{M^2} [\gamma^2 \rho_M(\gamma) + \int_0^\gamma d\gamma' \gamma'^2 \rho_M(\gamma')] &= \frac{1}{M^2} \left(\frac{d}{d\gamma} + 1 \right) \int_0^\gamma d\gamma' \gamma'^2 \rho_M(\gamma'). \end{aligned} \quad (32)$$

The extra term which appears in this relation is missing in the naive formula (12).

C. Short-time behavior

At the times shorter than τ_H , $\tau < 1$, an additional term arises,

$$\int_0^\gamma d\nu \Theta(\tau - 1 + \nu/\gamma) \nu^{M-1} e^{-\nu} = \varphi_{M-1}(\gamma) - \int_0^{(1-\tau)\gamma} d\nu \nu^{M-1} e^{-\nu}, \quad (33)$$

which depends on the time. As a consequence the decay law contains along with the smoothly distributed exponential contributions an additional polynomial term

$$\begin{aligned} K_M(\tau < 1) &= \int_0^\infty d\gamma e^{-\gamma\tau} \tilde{w}_M(\gamma) + \frac{1}{M} \frac{1 - (1-\tau)^M}{\tau^2} - \frac{1}{\tau} = \\ \int_0^\infty d\gamma e^{-\gamma\tau} \tilde{w}_M(\gamma) - \frac{1}{M} \sum_{m=0}^{M-2} (-1)^m \binom{M}{m+2} \tau^m. \end{aligned} \quad (34)$$

Formally, we still can present this formula in the form

$$K_M(\tau < 1) = \int_0^\infty d\gamma e^{-\gamma\tau} w_<(M, \gamma) \quad (35)$$

with a "distribution density" $w_<(M, \gamma) = \tilde{w}_M(\gamma) + w_M^{(sing.)}(\gamma)$. The price paid is the singular nature of the additional weight function

$$w_M^{(sing.)}(\gamma) = -\frac{1}{M} \sum_{m=0}^{M-2} (-1)^m \binom{M}{m+2} \frac{d^m}{d\gamma^m} \delta(\gamma), \quad M \geq 2. \quad (36)$$

This singular "distribution" differs in two important respects from that extracted from the Verbaarschot's asymptotic expansion: first, it contains only a finite sum of derivatives of the δ -function and, second, all singular terms are concentrated near $\gamma = 0$ rather than $\gamma = \gamma_W$.

We should stress that the both regular and singular parts play, generally, equally important roles. In particular,

$$K_M(0) = \frac{M+1}{2} - \frac{M-1}{2} = 1. \quad (37)$$

The first contribution comes from the regular and the second from the singular parts of the "distribution" $w_<(M, \gamma)$. They are of the same order of magnitude when $M \gg 1$.

Integration over γ gives now

$$K_M(\tau < 1) = \frac{1}{M} \frac{1 - (1 - \tau^2)^M}{\tau^2(1 + \tau)^M}, \quad K_M(0) = 1. \quad (38)$$

Combining all found results we arrive finally at

$$K_M(\tau) = \frac{1}{M} \frac{1}{\tau^2(1 + \tau)^M} [1 - \Theta(1 - \tau)(1 - \tau^2)^M], \quad 0 \leq \tau < \infty. \quad (39)$$

This function satisfies all necessary conditions and is continuous though not analytical in the point $\tau = \tau_W = 1$.

In the semiclassical limit $M \gg 1$ the characteristic decay time $\tau_W = 1/\gamma_W = 1/M \ll 1$ is much shorter than the Heisenberg time. The polynomial term is therefore of principal importance. It becomes obvious from the following equivalent presentation of the Eq. (38)

$$K_M(\tau < 1) = \frac{1}{M\tau^2} [e^{-M \ln(1+\tau)} - e^{M \ln(1-\tau)}] = e^{-M(\tau + O(\tau^3))} \frac{2 \sinh[\frac{M}{2}(\tau^2 + O(\tau^4))]}{M\tau^2} \approx e^{-M\tau} = e^{-\gamma_W \tau}. \quad (40)$$

Thus such an exponential semiclassical decay with the characteristic Weisskopf's decay rate γ_W cannot be directly traced to the statistics of resonance widths. Deviation from the exponential law due to the neglected terms becomes significant after the time $\tau_q \sim 1/\sqrt{M} \gg \tau_W$ [22, 23].

D. Mean cross section

The mean cross section can be expressed in two equivalent forms

$$\langle \sigma \rangle = \frac{1}{M} = \int_0^1 d\tau K_M(\tau) + \int_1^\infty d\tau K_M(\tau) = \int_0^\infty \frac{d\gamma}{\gamma} e^{-\gamma} \left[P_{(M-1)}(\gamma) - 1 + \frac{\gamma}{M} \right] + \int_0^\infty w_M^{(sing.)}(\gamma) \frac{1-e^{-\gamma}}{\gamma}. \quad (41)$$

The long-time contribution ($\tau > 1$) rapidly decreases when the number of channels M grows. Even for $M = 1, 2$ it amounts only to 30% and 10% respectively. When $M \gg 1$ the following asymptotic expansion

$$\int_1^\infty d\tau K_M(\tau) = K_M(1) \frac{2}{M} \int_0^{M/2} d\xi e^{-(\xi+3\xi^2/2M+\dots)} \approx \frac{2}{M^2} e^{-M \ln 2} \quad (42)$$

shows that this contribution diminishes very fast. The mean cross section is defined by the short time evolution.

On the other hand, the contributions of the regular and singular parts of the decay rates distribution are equally important. Indeed

$$\int_0^\infty \frac{d\gamma}{\gamma} e^{-\gamma} \left[P_{(M-1)}(\gamma) - 1 + \frac{\gamma}{M} \right] = \frac{1}{M} + \sum_{m=2}^M \frac{1}{m}. \quad (43)$$

The logarithmically growing extra term is perfectly compensated by the contribution of the singular part.

VI. CONCLUSIONS

The decay law $K(t)$ of a complicated unstable state of a classically chaotic system can be formally presented in the form of a Laplace integral as a weighted mean value of the decay exponents. By this the notion of the decay rates distribution is introduced irrelative of statistics of the resonance poles of the scattering amplitudes. The connection of this distribution with statistics of the resonance widths is then investigated. Exact analytically solution is found in the case of systems with broken time-reversal symmetry. It is demonstrated that only the long-time $\tau \gg \tau_H = 1$ asymptotic behavior of the decay law is governed by the statistics of the resonance widths. For the shorter times $\tau < \tau_H = 1$ the contributions of the connected to the widths statistics smooth part and the singular part of the "distribution" $w_<(M, \gamma)$ are equally important. In particular, the approximately exponential semiclassical decay with the characteristic Weisskopf's decay rate γ_W results from interrelation of the both of them.

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